Diode AM Conversion Efficiency in Crystal Sets and Some Implications

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(Note: Because of the incompatibilities of most internet browsers with many special characters here a short conversion table of replaced symbols:
OM = capital Omega, ***om*** = low-case Omega, ***SQ.RT*** = square root, ***eta*** = greek character, ^ = to the power of .)

This is an attempt to obtain a theoretical understanding of diode conversion efficiency in xtal sets. It was motivated by the wish to interpret measured values and to obtain an explanation for the experimentally observed increase in audio power with growing tank-circuit Q at weak RF signals. We assume here that the tank-circuit Q is increased by replacing the existing tank inductor with one having a higher unloaded Q. It turns out that the audio power is caused by a conversion-efficiency increase as a result of the higher RF voltage across the tank.

We first determine the detected current Id if an unmodulated RF voltage of angular frequency ***OM***, namely

V = E * sin ***OM***t,   (1)

is applied to the detector diode. This voltage must be inserted in the static diode characteristic

Id = Is*[exp(Vd/(n*26 mV)] - Is   (2)

at room temperature (300 K), where Is denotes the diode saturation current and n the ideality factor. For most Schottky diodes n is 1.05.

To simplify matters we here restrict the case to small-signal operation which anyway is the range of interest when dealing with weak DX stations. The square-law region of the rectifier characteristic, comprising the diode and its load, is considered to represent this small-signal range. As shown below, in the square-law region an increase in detected current is proportional to the square of the amplitude of the RF voltage applied. At larger signal levels the output current grows only linearly with the RF voltage (linear region) - No external DC diode bias is here assumed to be used.

As customary practice for small voltages, Eq. (2) is expressed by a Taylor series where derivations up to only the second (quadratic) term are considered, so that one obtains

Id = f ´(0)*V + 0.5*f ´´(0)*V^2   (3)

where

f ´(0) = Is / n*26mV

denotes the first derivative of Eq. (2) at zero bias (original working point), thus representing the slope of the characteristic at this point. The term

f ´´(0) = Is / (n*26mV)^2

is the second derivative that describes the curvature.

We now insert the sine function for the RF voltage, Eq. (1), in Eq. (3) and first solve for the time-invariant term which becomes

$$Idc = f''(0) \cdot E^2 / 4. \qquad (4)$$

Eq. (4) represents the detected DC current in case of zero load, obtained because of the squared sine voltage function. We now assume that a resistive load R is provided which shall be shunted for the RF by a suitable capacitor, as is usual practice in rectifier/demodulator circuits. Then Eq. (4) can be shown to change to

$$Idc = f'' \cdot E^2 / (4 + 4 \cdot f' \cdot R). \qquad (5)$$

This current leads to the DC voltage that we observe across a diode under RF operation. Eq. (5) can be interpreted as being Ohm´s Law for the diode detector, where $f'' \cdot E^2 / 4 \cdot f'$ represents the internal EMF of the detector, and $1/f' = (n \cdot 26mV / Is) = Rdo$ the internal (diode) resistance that is in series with the external load R.

Now we assume that the applied RF voltage is sinusoidally amplitude-modulated by an audio signal of angular frequency ***om***. In this case Eq. (1) becomes

$$Vd = [E\,(1 + m \cdot \sin \boldsymbol{om}\, t)] \cdot \sin \boldsymbol{OM}\, t \qquad (6)$$

where m represents the modulation factor, and the expression within the brackets [...] is now the amplitude of the RF voltage. If we substitute this amplitude, that carries the audio information, instead of the previous constant voltage E in Eq. (5), dissolve the brackets, and extract the current component at the basic audio frequency ***om*** we obtain

$$Ia = f'' \cdot m \cdot E^2 / [2 \cdot (1 + f' \cdot R)]. \qquad (7)$$

If the load is not purely resistive we can for R insert the (amount of the) impedance Za at the audio frequency ***om***. In case of impedance matching between the (dynamic or differential) diode resistance Rdo (= $1/f'$) at zero volts and the audio load R or Za, resp., Eq. (7) simplifies to

$$Ia = f'' \cdot m \cdot E^2 / 4. \qquad (7a)$$

For the audio output power we then obtain by use of Eq. (7a)

$$Pa = Za \cdot 0.5 \cdot Ia^2 = Za \cdot 0.5 \cdot (f'')^2 \cdot m^2 \cdot E^4 / 16. \qquad (8a)$$

The term 0.5 is caused by the fact that the current Ia in Eqs. (7,7a) is the peak value, but for calculating the power we have to use the RMS value squared. If we introduce the RF power at the diode, namely $Pr = Erms^2 / Za$, then Eq. (8a) for Pa becomes

$$Pa = Za^3 \cdot (f'')^2 \cdot m^2 \cdot Pr^2 / 8. \qquad (8b)$$

The diode conversion efficiency ***eta*** shall be defined as the demodulated audio power Pa in relation to the audio (AM) sideband power Psb that is contained in the RF power Pr:

$$\boldsymbol{eta} = Pa / Psb. \qquad (9)$$

With Pa being proportional to Pr^2 and Psb to Pr, the efficiency ***eta*** changes linearly with the RF power Pr. Instead of the efficiency ***eta*** the conversion loss Lc (in dB) is sometimes quoted, namely

$$Lc = 10 \log (1/\boldsymbol{eta}). \qquad (10)$$

The required sideband power can be shown to be

$$Psb = Pr*m^2 / (2 + m^2), \qquad (11)$$

which for example yields Psb = 0.17*Pr (17 %) if the modulation factor is m = 0.65. (See footnote 1).

The expressions derived above for the audio power and the conversion efficiency are valid only in the square-law region of the rectifier. So we must know up to what critical input power this region extends before the transition range to the linear region is gradually to start. In the linear region the efficiency is high and remains constant to first order since both Pa and Psb grow in proportion to Pr. A pertinent calculation, as outlined in footnote 2, shows that the critical input power depends on the impedance level (tank resonance resistance Ro). Increasing (lowering) the impedance by a factor of N causes to lower (increase) the critical input-power point by also this factor of N. As a numerical example (from which values for other impedances can be derived), the calculation for an assumed Ro = 500k ohms indicates that the true square-law region ends when the RF power at the diode approximately reaches Pr = 1.3 nW. In case of the here assumed matching there are 3 dB of the input RF power being dissipated in the resonance resistance Ro. Adding then 3 dB to account for this additional loss, a set with an Ro = 500k ohms can be operated at up to 2.6 nW of RF power before the above Eqs. (7) to (11) are no longer valid.

<u>Numerical Examples</u>

We now consider an xtal set where a Schottky diode (n = 1.05) is connected to the top of the tank circuit and matched to it. As advocated in particular by Ben Tongue (ref. 1), the headphones are connected to the diode via an audio transformer, the input impedance of which is matched to the diode resistance Rdo. The DC load of the diode is made equal to Rdo as well.

(A)
First we investigate the case for a tank circuit of relatively low Q, namely of an unloaded Qo = 150. Such Q values are typical for conventional tank circuits that employ coils (cylindrical, spiderweb, basket, etc.) wound with solid wire. At 1 MHz and assuming a tank inductor of 0.2 mH the tank resonance resistance Ro then is 188k ohms. An RF power of the critical value at the end of the square-law region shall be applied to the diode: Derived from the above quoted example for Ro = 500k ohms the critical power of Pr now is (500/188)*1.3 nW = 3.46 nW. For this Pr, an Is of 145 nA in case of Ro = 188k ohms, and assuming m = 0.65, we obtain by use of Eq. (8b) an audio power of Pa = 0.159 nW. According to Eq. (11) the sideband power is Psb = 0.59 nW which then leads to a diode conversion efficiency of 0.27 (27 %).

(B)
Let us now investigate how the audio power and the efficiency change when the RF power is

lowered. The minimum audio power that just produces an audible signal in my sound-powered phones (WE Deck Talker) is 0.5 pW, i.e. they have a sensitivity of -93 dBm. To obtain a Pa of 0.5 pW at the phones and accounting for an audio-transformer loss of 1.5 dB in my case, it then needs according to Eq. (8b) an RF power at the diode of 0.275 nW. The set at such must be provided with 3 dB more (tank loss), hence with about 0.55 nW of power from antenna/ground. At this lower-limit RF power the diode conversion efficiency has after Eq. (9) dropped to 0.015 (1.5 %). Compared to the case (A) the efficiency is lower by a factor of 18, thus by the factor of the reduction in RF power. Physically the low efficiency is a result of the diminishing difference between forward and reverse diode resistances at small RF signals, where both of them approach Rdo that we took as the mean value in the impedance-matching considerations.

(C)
Then the influence of tank-circuit Q on audio power and diode conversion efficiency shall be investigated, at constant RF power Pr. If the tank resonance resistance Ro is altered because of a change in Qo, the Rdo and Za change likewise (matching requirement). The term (f ´´)^2 in Eq. (8b) is proportional to (1/Rdo)^2 which multiplied by Za^3 (Za = Rdo) causes the audio power Pa to change in proportion to Za and thus to the Qo change. And accordingly the conversion efficiency, Eq. (9), is changed by the same factor.

In the set I used for the 2003 DX Contest of the U.S Xtal Set Society I employed special ferrite- core coils (see ref. 2; also ref. 3) that provide tank Qo values of 450 at 1 MHz. With a tank inductor of 0.2 mH the resonance resistance of the tank circuit is then Ro = 565k ohms, requiring a diode of Is = 48 nA. In this case the critical input power (end of square-law region) occurs at (500/565)*1.3nW = 1.15 nW. It is three times lower than the value obtained for a tank Qo of 150 as in (A). The beneficial linear detector region, with its higher conversion efficiency, has now moved to lower input-power values. At an RF power of 0.275 nW, as was required in case (B) for reaching the threshold of the phones, we now obtain three times more audio power, namely 1.5 pW. The minimum detectable RF power drops by a factor of ***SQ.RT*** 3 = 1.732 to (0.275 nW / 1.732) = 0.158 nW. This means that 0.158 nW + 3 dB = 0.316 nW must be supplied to the set by the antenna.

Experimentally I find the minimum detectable RF power to be about 3 dB higher than the calculated 0.55 nW in (B) and the 0.316 in (C). This discrepancy seems mostly to be caused by the fact that in the experiment the RF power was determined from two measurements of the RF voltage across the tank circuit (first in the unloaded case for obtaining Qo and Ro, secondly under operation for getting the threshold Pr by use of Ro). In both measurements the set-up tends to additionally load the tank circuit and so to lead to an apparent reduced sensitivity. In addition some loss will occur as a result of the non-perfect matching conditions in the real set. Allowing for these losses, the calculations and the measurements can be said to be in reasonable agreement.

Concluding Remark

I think the above calculations provide a convincing quantitative explanation on how and why the audio power (sensitivity) of a set increases in the small-signal range with growing tank Q, at constant available RF power. As mentioned, this effect is limited to operation in the square-law region. The low efficiency experienced in that region is improved by the higher RF voltages developed across a tank circuit of high Q (high resonance resistance). Tank circuits with carefully designed coils can in the BC band be made to reach Qo values of 1000; see ref. 1 (paper # 26) and ref. 4. This is about the upper limit for the Q before noticable audio impairment is experienced, because of the loaded -3dB bandwidth then dropping to 3 to 4 kHz. Compared to 150, as about the unloaded tank Qo found in many average xtal sets, going to a Qo of 1000 increases the audio power by a factor of 6.66 (ca. 8.2 dB) and lowers the detection threshold by the factor of ***SQ.RT*** (6.66) = 2.58.

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Footnotes:

1. AM stations mostly use amplitude compression which enhances the low-amplitude audio portions. According to station engineers a modulation factor of m = 0.65 is a good mean value for averaged music and speech contributions if amplitude compression and a peak modulation of m = 1 are employed.

2. The DC output current of Schottky-diode detectors is here calculated as a function of RF input voltage by employing Bessel functions; see e.g. Ref. 5. Such a calculation is not restricted to the square-law region but also covers the linear region including the transition range between them, the latter extending over about 15 dB of input voltage (power). The Bessel-function approach is however not suited to obtain the audio power Pa as easily as it was possible above in deriving Eqs. (8a), (8b). But the approach allows us to determine the input power at which the output is no longer proportional to the square of the input. The position of this limiting (critical) input-power point is the same for an unmodulated and for a modulated RF input signal.
To keep the maths simple I decided to calculate the DC output current for zero load and then to graphically add, at the same current values, the load voltages as obtained from a drawn load line. In this way the combined characteristic curve of the loaded detector could be determined. For zero load the DC current has the straightforward form of

$$Idc = Is * Io(E/n*26mV) - Is,$$

where Io(...) is the modified Bessel function of zero order - see tabulated values - and E the amplitude of the RF voltage at the diode. Up to a diode voltage of about 1*(n*26mV) the zero-load DC current according to Eq. (4) is in good agreement with the more accurate one derived here by employing the Bessel function. The limit of the square-law region was determined by inspecting the curve of the combined characteristic for the point where the DC output current tends to no longer increase as the square of a change in input voltage. It turns out that the approximate end of the square-law region (critical point) is reached when the RF voltage across the diode has grown to 1*(n*26mV), independent of the RF impedance level. Since the changes in the characteristic curve are smooth and gradual there is a tolerance in determining the "point" (tolerance of about 1.5 dB). The results obtained here are in fair agreement with those presented in Ref. 1 (paper # 15A) as derived from SPICE simulations in case of the particular impedance value of 700k ohms considered there.

References:

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